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Testing Bell's Inequality with Polarization-Entangled Photons

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Abstract

In this experiment, we generated polarization-entangled photon pairs using spontaneous parametric down-conversion in a pair of orthogonally oriented Type-I BBO crystals. By performing a CHSH Bell test with four combinations of linear polarizer angles, we measured the strength of the two-photon correlations and evaluated the Bell parameter S. Over 40 independent trials, every measurement yielded $S > 2$, violating the classical local-realistic bound with high statistical significance. The average value, $\overline{S} = 2.337 \pm 0.011$, demonstrates a consistent and robust Bell violation and confirms the presence of non-classical entanglement in the photon pairs produced by the quED system. Although experimental imperfections prevented us from reaching the ideal quantum limit of $S = 2\sqrt{2}$, the magnitude and stability of our measured correlations agree well with the expectations for a realistic SPDC-based entangled photon source. These results highlight how entangled states exhibit correlations that are fundamentally incompatible with classical physics, a key idea underlying modern quantum information science.

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1 Introduction

Quantum entanglement is one of the most surprising and non-classical predictions of quantum mechanics. Two particles can be prepared in a shared quantum state where the outcome of a measurement on one particle is strongly correlated with the outcome on the other, even when the particles are far apart. These correlations cannot be explained by any classical picture based on local hidden variables, and they serve as a key resource for quantum information technologies such as quantum communication and quantum cryptography.

In this experiment, we use the quED (Quantum Entanglement Demonstrator) to generate pairs of polarization-entangled photons and measure their correlations. The quED uses a nonlinear optical process to produce photon pairs whose polarizations are strongly linked, allowing us to directly study how entangled particles behave under different measurement settings. By analyzing coincidence detections between the two photons, we can observe the polarization correlations and compare them to the predictions of classical and quantum models.

A major goal of the experiment is to test the CHSH form of Bell's inequality, which sets an upper limit on the strength of correlations achievable by any classical local-realistic theory. Quantum mechanics predicts stronger correlations for entangled states, leading to a measurable violation of the inequality. By recording coincidence counts for different combinations of polarizer angles, we compute the CHSH parameter S. Observing $S > 2$ provides clear evidence of entanglement and demonstrates behavior that cannot be explained by classical physics, which is precisely the phenomenon this experiment is designed to reveal.

We also measure full correlation curves by scanning the angle of one polarizer, which allows us to visualize how the correlations vary across different measurement settings. These measurements give us both qualitative and quantitative insight into the non-classical nature of the entangled state generated by the quED system.

2 Theory

We will now provide a deeper discussion of the theory involved in this experiment. The main concepts at play for this experiment are: Spontaneous Paramentric Down-Conversion (SPDC), Formation of the Bell State, Polarization Measurement and Coincidence Detection, Correlation Coefficients, the CHSH Inequality, and the Significance of the Bell State Violation.

2.1 Spontaneous Parametric Down-Conversion (SPDC)

The source of entangled photons in this experiment is spontaneous parametric down-conversion (SPDC), a nonlinear optical process in which a pump photon inside a nonlinear crystal occasionally converts into a pair of lower-energy photons. Energy and momentum conservation restrict the possible emission directions and wavelengths, causing the down-converted photons to emerge on well-defined cones. Although SPDC is a probabilistic process, the two photons are created simultaneously in the same interaction, which leads to strong correlations in their polarization and time of arrival.

In the quED setup, entanglement is generated using two thin Type-I BBO crystals placed back-to-back, with their optical axes rotated by 90°. The first crystal converts pump photons into horizontally polarized photon pairs ($|HH\rangle$), while the second produces vertically polarized pairs ($|VV\rangle$). When the pump is polarized at 45°, both crystals emit with equal probability.

2.2 Formation of the Bell State $|\Phi^+\rangle$

For these two processes to form an entangled state, there must be no physical way to determine which crystal produced a given photon pair. The quED accomplishes this by carefully overlapping the spatial modes of the two emissions and by using birefringent compensation crystals to erase timing differences between the $|HH\rangle$ and $|VV\rangle$ components.

When this indistinguishability between crystals is achieved, the output state is the coherent superposition

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right),$$

one of the four maximally entangled Bell states. The key feature of this state is that the photons are perfectly correlated in any linear polarization basis: if one photon is measured to be aligned with some angle α, the other photon will be found aligned with the same angle $\beta = \alpha$ with high probability.

2.3 Polarization Measurement and Coincidence Detection

Each photon travels into a separate detection arm containing a rotatable polarizer and a single-photon detector. A coincidence event occurs when both detectors click within a short time window, indicating that both photons from a down-converted pair were detected.

For the $|\Phi^+\rangle$ state, the joint probability that both photons pass through linear polarizers at angles α and β is

$$P(\alpha, \beta) = \frac{1}{2}\cos^2(\alpha - \beta).$$

This $\cos^2$ dependence produces the sinusoidal correlation fringes observed when scanning one polarizer. Classically correlated states do not exhibit such high-visibility fringes, making these curves a clear signature of quantum entanglement.

2.4 Correlation Coefficients $E(\alpha, \beta)$

To quantify the strength of the correlations, we compute the normalized correlation coefficient

$$E(\alpha, \beta) = \frac{C_{\text{same}} - C_{\text{diff}}}{C_{\text{same}} + C_{\text{diff}}},$$

where C_{same} includes coincidence counts where the photons either both pass or both are rejected by their polarizers, and C_{diff} includes coincidences where one photon passes and the other is rejected. Experimentally, this corresponds to four specific coincidence measurements with polarizers set to (α, β), $(\alpha, \beta^{\perp})$, $(\alpha^{\perp}, \beta)$, and $(\alpha^{\perp}, \beta^{\perp})$, where $\perp$ denotes a rotation by $90°$.

For the $|\Phi^+\rangle$ state, theory predicts

$$E(\alpha, \beta) = \cos(2(\alpha - \beta)) .$$

2.5 The CHSH Bell Inequality

The CHSH inequality provides a way to distinguish classical local-realistic models from quantum predictions. In the CHSH scenario, each photon is measured with one of two possible polarizer angles. Alice chooses between α and α', while Bob chooses between β and β'. From these four combinations, the CHSH parameter is defined as

$$S = E(\alpha, \beta) + E(\alpha', \beta) - E(\alpha, \beta') + E(\alpha', \beta').$$

Any theory based on local hidden variables must satisfy

$$|S| \leq 2.$$

However, for a maximally entangled state such as $|\Phi^+\rangle$, quantum mechanics predicts a maximum value of

$$S_{\text{QM}} = 2\sqrt{2} \approx 2.828.$$

Thus, if the experimentally measured value satisfies $S > 2$, we have directly violated the Bell inequality and demonstrated that the correlations in our data cannot be explained by any classical local-realistic model.

2.6 Significance of the Bell Violation

A Bell violation is one of the most direct and compelling demonstrations of non-classical behavior in quantum mechanics. Unlike many experiments that require complex interpretation, the CHSH test compares the measured correlations against a strict classical bound. Observing $S > 2$ is therefore strong evidence that the quED system is generating genuine quantum entanglement rather than classical or statistical correlations.

3 Methods

3.1 Experimental Setup

The experiment was performed using the quED (Quantum Entanglement Demonstrator) system, which generates polarization-entangled photon pairs via spontaneous parametric down-conversion. A 405 nm pump laser was directed into two adjacent Type-I BBO crystals with orthogonal optical axes. The pump polarization was set to 45° so that horizontally polarized ($|HH\rangle$) and vertically polarized ($|VV\rangle$) photon pairs were produced with equal probability. Birefringent compensation crystals were used to remove relative timing differences between the two processes, allowing the output state to approximate the Bell state $|\Phi^+\rangle$.

The down-converted photons were sent into two separate detection arms. Each arm contained a rotatable linear polarizer and a single-photon avalanche photodiode (APD). Coincidence events were recorded using the quED coincidence electronics within a fixed coincidence window.

3.2 Alignment Procedure

The optical setup was carefully aligned to maximize coincidence counts and ensure high-quality entanglement. The pump beam was first centered through the BBO crystals to optimize down-conversion efficiency. Steering mirrors and fiber couplers in each detection arm were then adjusted to maximize singles counts. Final alignment was achieved by iteratively adjusting the mirrors to maximize the coincidence rate, indicating optimal spatial overlap and indistinguishability of the photon pairs.

3.3 Measurement of Correlation Curves

To observe polarization correlations, one polarizer was fixed at a chosen angle α while the other polarizer was scanned from 0° to 180° in increments of typically 10°. At each angle pair (α, β), the quED control unit recorded the singles counts on each detector and the corresponding coincidence counts.

These measurements were used to generate correlation curves of the form

$$P(\alpha, \beta) \propto \cos^2(\alpha - \beta),$$

which visually indicate the presence of polarization entanglement.

3.4 Data Collection

At each setting of the polarizer, singles counts and coincidence counts were recorded.

To perform the CHSH Bell test, coincidence measurements were taken at the standard angle settings

$$\alpha = 0^\circ, \quad \alpha' = 45^\circ, \quad \beta = 22.5^\circ, \quad \beta' = 67.5^\circ.$$

For each angle pair, four coincidence measurements were used to compute the correlation coefficient

$$E(\alpha, \beta) = \frac{C_{\text{same}} - C_{\text{diff}}}{C_{\text{same}} + C_{\text{diff}}}.$$

The CHSH parameter S was then calculated from the four correlation coefficients and compared to the classical bound $|S| \leq 2$.

3.5 Evaluation of the Bell Inequality

The experimentally obtained value of S was then compared with the classical bound $|S| \leq 2$. A value significantly above 2 indicates a violation of the CHSH inequality and provides direct evidence of non-classical entanglement between the photon pairs.

4 Results

We performed 40 independent CHSH measurements. In each run, the CHSH parameter S was computed from coincidence counts measured at the four standard polarizer angle combinations. All 40 measurements yielded $S > 2$, indicating a violation of the classical CHSH bound in every trial.

Across all runs, the measured values ranged from

$$S_{\min} = 2.264 \quad \text{to} \quad S_{\max} = 2.536,$$

with typical single-run uncertainties of approximately ± 0.09. The unweighted mean over all measurements is

$$\overline{S} = 2.337 \pm 0.011,$$

where the quoted uncertainty is the standard error of the mean. This value lies approximately 32 standard deviations above the classical bound $S = 2$, demonstrating a statistically robust Bell violation.

Because the reported uncertainties were similar across runs, a weighted average was also computed using weights $w_i = 1/\sigma_i^2$. The weighted mean,

$$\overline{S}_w = 2.339 \pm 0.014,$$

is consistent with the unweighted result, indicating no significant run-to-run systematic variation.

Figure 1 shows the measured S values as a function of run number. The data fluctuate randomly about the mean with no observable drift over time, suggesting stable alignment and source performance throughout the measurement sequence. The histogram of S values shown in Figure 2 is approximately Gaussian and centered well above the classical threshold at $S = 2$. Figure 3 summarizes all 40 measurements with statistical error bars and highlights the clear separation between the experimental data and the classical bound.

Together, these results confirm that the quED system consistently produced polarization-entangled photon pairs exhibiting strong non-classical correlations.

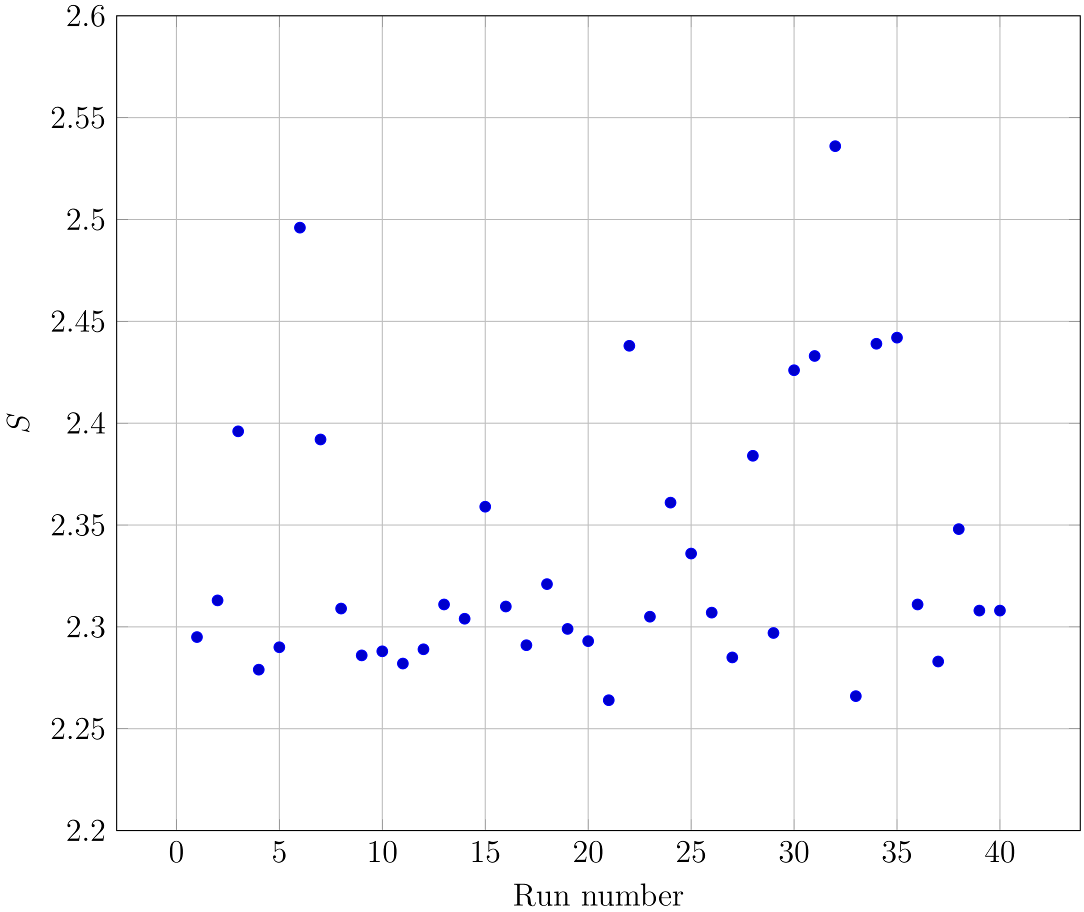

Figure 1: Measured CHSH parameter S as a function of run number.

Table 1: Measured CHSH parameter S and statistical uncertainty for each run. Each run used a 1 s integration time.

Run	S	Uncertainty σ_S
1	2.295	0.088
2	2.313	0.090
3	2.396	0.088
4	2.279	0.086
5	2.290	0.090
6	2.496	0.084
7	2.392	0.087
8	2.309	0.089
9	2.286	0.089
10	2.288	0.090
11	2.282	0.088
12	2.289	0.089
13	2.311	0.086
14	2.304	0.086
15	2.359	0.087
16	2.310	0.090
17	2.291	0.089
18	2.321	0.089
19	2.299	0.087
20	2.293	0.089
21	2.264	0.087
22	2.438	0.083
23	2.305	0.088
24	2.361	0.088
25	2.336	0.088
26	2.307	0.089
27	2.285	0.093
28	2.384	0.086
29	2.297	0.090
30	2.426	0.085
31	2.433	0.085
32	2.536	0.084
33	2.266	0.089
34	2.439	0.086
35	2.442	0.090
36	2.311	0.094
37	2.283	0.088
38	2.348	0.087
39	2.308	0.088
40	2.308	0.088

Table 2: Summary statistics for the CHSH measurements over 40 runs.

Quantity	Value
Number of runs N	40
Minimum S_{min}	2.264
Maximum S_{max}	2.536
Unweighted mean $\overline{S}$	2.337
Sample standard deviation σ_S	0.067
Standard error of the mean $\sigma_{\overline{S}}$	0.011
Weighted mean $\overline{S}_w$	2.339
Uncertainty of weighted mean	0.014
Classical CHSH bound	2.000
Ideal quantum value $2\sqrt{2}$	2.828

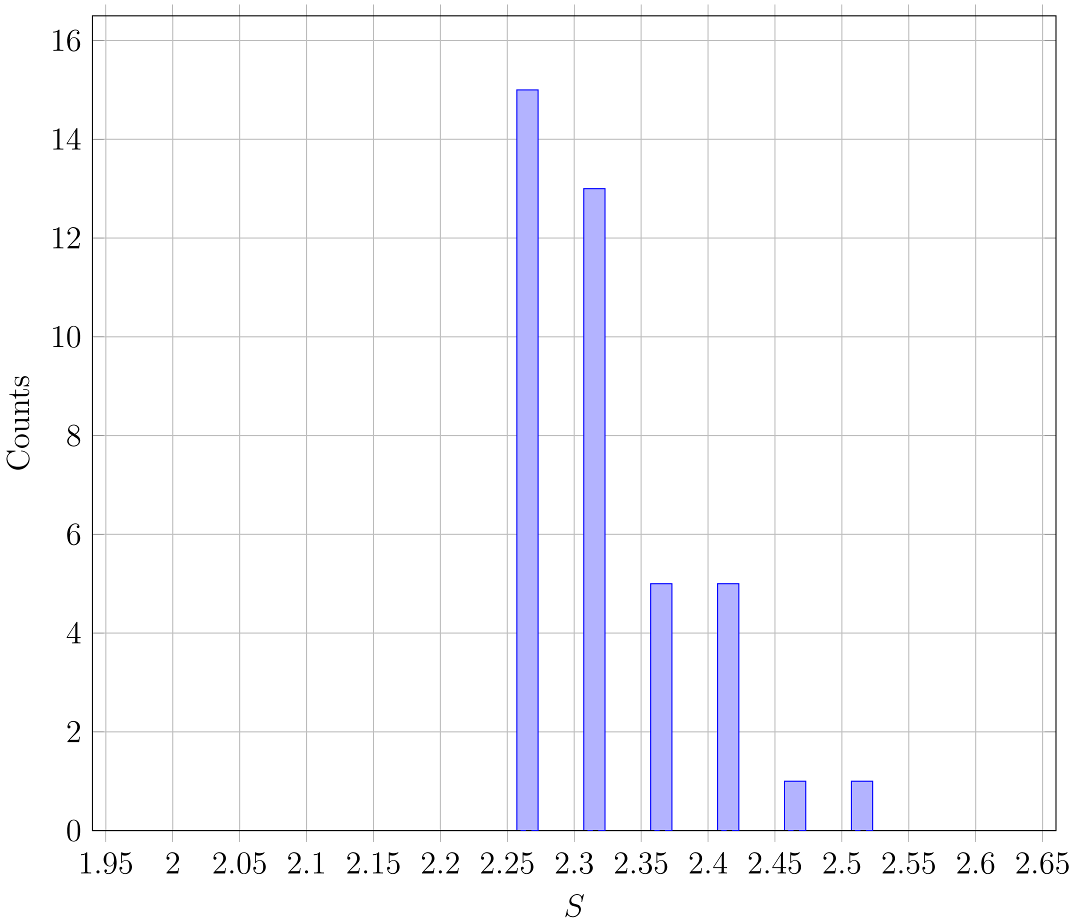

Figure 2: Histogram of the measured CHSH parameter S over 40 runs. Bin centers are shown on the horizontal axis.

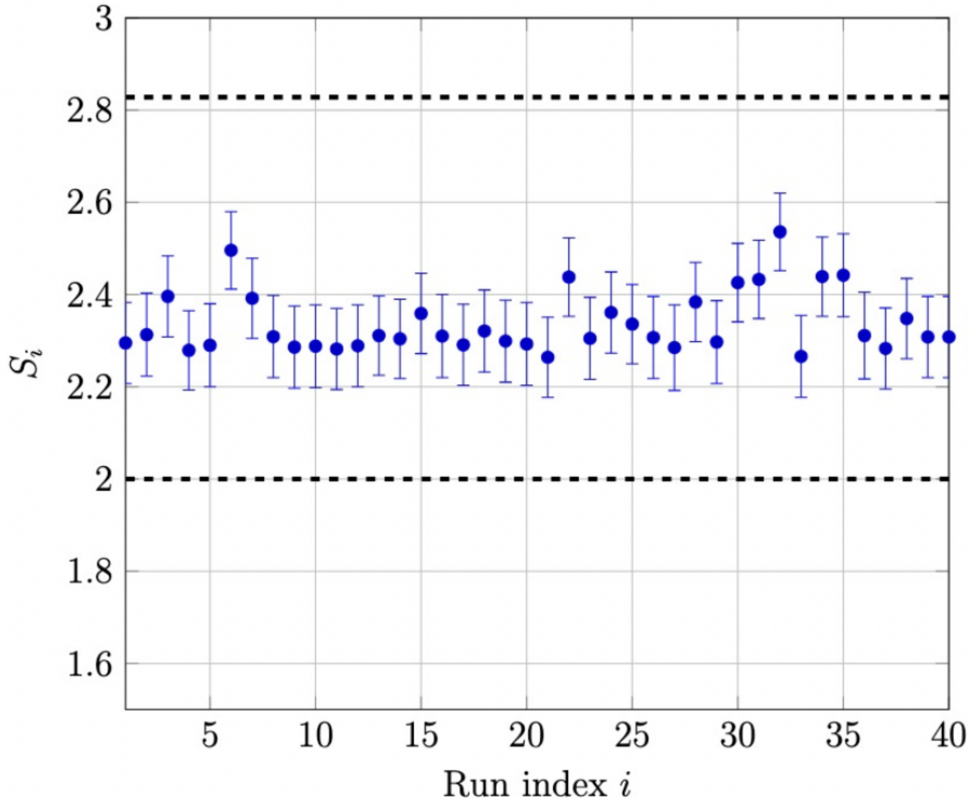

Figure 3: Measured CHSH values S_i for 40 independent trials, each shown with one–sigma statistical error bars. The lower dashed line marks the classical CHSH bound $S = 2$, while the upper dashed line indicates the quantum mechanical limit for a maximally entangled state, $S = 2\sqrt{2}$. The data fluctuate around a stable mean near $S \approx 2.33$ with no significant drift, and all 40 values lie well above the classical threshold, demonstrating a clear violation of the CHSH inequality.

5 Discussion

The CHSH measurements performed in this experiment provide clear evidence of non-classical correlations between the photon pairs produced by the SPDC source. In all 40 trials, the measured value of the Bell parameter satisfied $S > 2$, which cannot be explained by any classical local-realistic model. This consistent violation demonstrates that the photon pairs generated by the quED system were genuinely entangled.

The average value, $\overline{S} = 2.337 \pm 0.011$, lies well above the classical bound but below the ideal quantum prediction of $S = 2\sqrt{2}$. This difference is expected in a realistic laboratory setting. In practice, no experimental system is perfectly isolated or perfectly aligned, and small imperfections in the optical setup and detectors reduce the strength of the measured correlations. As a result, the observed Bell violation is strong but not maximal.

An important feature of the data is its stability over time. The measured S values fluctuate around a constant mean with no systematic drift across runs, indicating that the entangled photon source and measurement apparatus remained stable throughout data collection. The spread in values is consistent with normal statistical fluctuations rather than slow changes in alignment or source performance. Occasional runs with higher S values likely correspond to moments when the optical alignment was especially well optimized.

One practical challenge encountered during the experiment was the time required for careful optical alignment. Because the quED system is largely open to the surrounding laboratory environment, ambient light and reflections can make alignment more difficult and increase background noise. A straightforward improvement would be to enclose the optical path during alignment. Reducing stray light would improve visibility of alignment beams, lower background counts, and likely shorten alignment time while improving repeatability.

Overall, the results of this experiment clearly demonstrate the non-classical nature of entangled photon pairs and provide a direct experimental test of Bell's inequality. The observed violations highlight the fundamental departure of quantum mechanics from classical intuition and illustrate how carefully designed measurements can reveal this behavior in a laboratory setting.

6 Conclusion

In this experiment, we generated polarization-entangled photon pairs using a Type-I SPDC source and tested their correlations through a CHSH Bell inequality measurement. By measuring coincidence rates for four combinations of polarizer angles, we obtained a clear and statistically robust violation of the classical bound $S \leq 2$. Across 40 independent trials, every measurement exceeded this limit, with an average value of $\overline{S} = 2.337 \pm 0.011$.

The consistency of the measured S values over time indicates stable source performance and reliable alignment throughout data acquisition. Although experimental imperfections prevented us from reaching the ideal quantum value $S = 2\sqrt{2}$, the observed violations are fully consistent with quantum mechanical predictions for a realistic laboratory entanglement source.

Overall, this experiment provides a direct demonstration of quantum entanglement and illustrates how Bell-type measurements reveal correlations that cannot be explained by any classical local-realistic theory. The results highlight both the fundamental nonclassical nature of entangled states and the practical considerations involved in generating and characterizing entanglement in quantum optics experiments.

References

1. qutools GmbH, *quED User Manual*. Available at: `https://www.qutools.com/quED`.